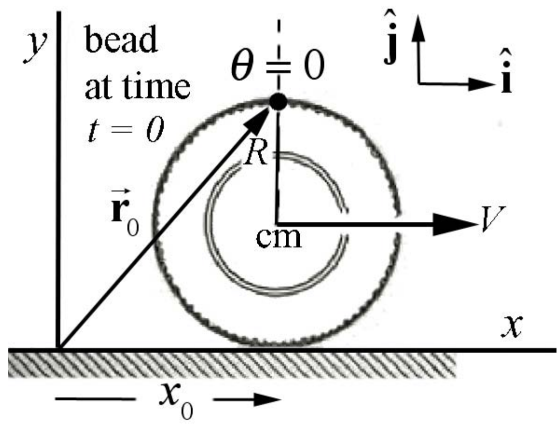
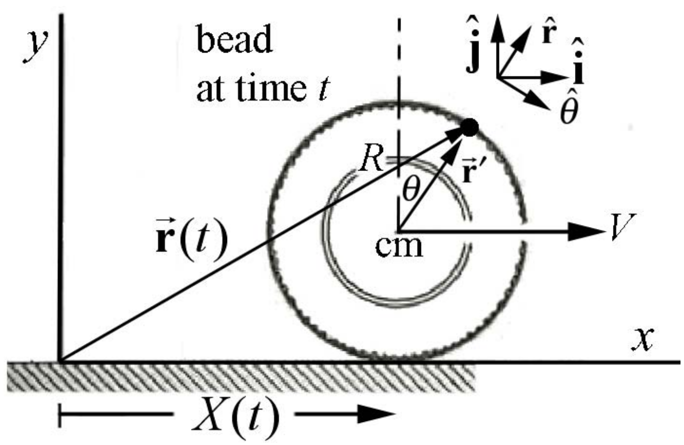
# Problem Set 12

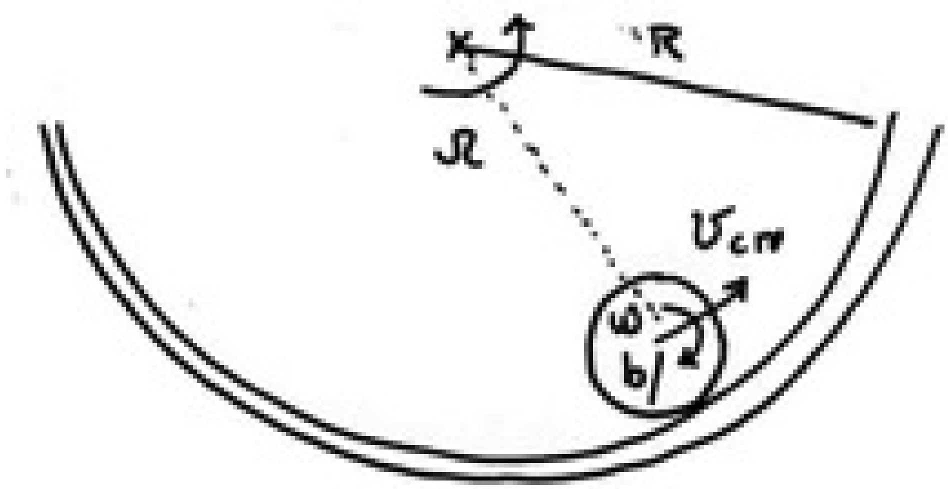
1. **Bicycle Wheel** A bicycle wheel of radius $R$ is rolling without slipping along a horizontal surface. The center of mass of the bicycle is moving with a constant speed $V$ in the positive $x$-direction. A bead is lodged on the rim of the wheel. Assume that at $t = 0$, the bead is located at the top of the wheel at $x(t = 0) = x_0$, $y(t = 0) = 2R$.




What are the $x$- and $y$-components of the position of the bead as a function of time according to an observer fixed to the ground? Express you answer in terms of some or all of the following: $x_0$, $V$, $t$ and $R$.

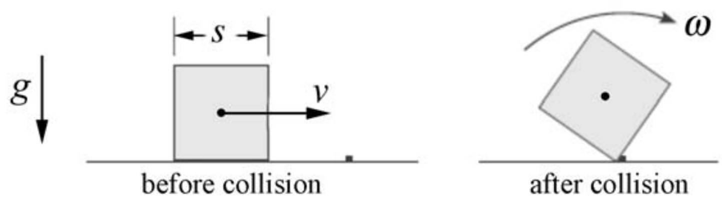
2. **Rolling Without Slipping**




A ball of radius $b$ and mass $M$ is rolling without slipping on the surface of a ring of radius $R > b$. The angular speed of the ball is $\omega$. Suppose the ring is rotating with angular speed $\Omega$ as shown in the figure and the ball is rolling without slipping. What is the speed of the center of mass of the ball? Write your answer using some or all of the following: b, R, $\omega$ and $\Omega$.

3. **Cubical Block Collision with Low Ridge**

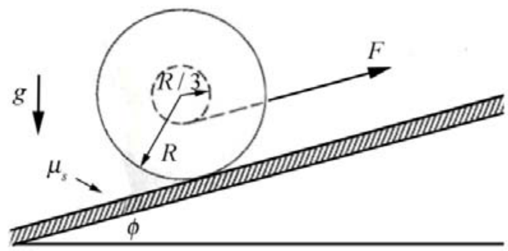
   A uniform cubical block of mass $m$ and edge length $s$ slides without friction with speed $v$ on a horizontal surface. It collides inelastically with a low ridge on the surface. Assume the collision is instantaneous. It subsequently pivots without friction about that ridge. The moment of inertia of a cube about an axis normal to and centered on one of its faces (and thus passing through the center of mass) is $ms^2/6$. The downward acceleration of gravity is $g$.




   (a) Find the initial rotation rate of the cube $\omega_a$ just after the collision. Express your answer in terms of some or all of the parameters $m$, $s$, $v_0$ and $g$.

   (b) Find the maximum value $v_0$ such that the cube does not topple over and end up resting on other side of the ridge. Express your answer in terms of some or all of the parameters $m$, $s$, $v_0$ and $g$.

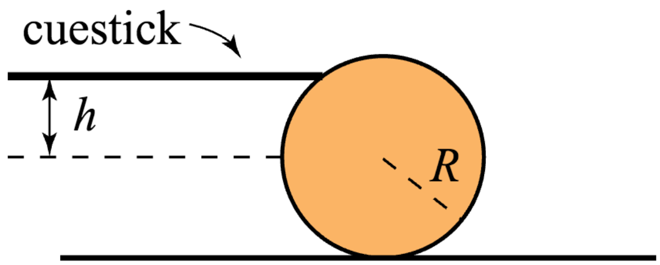
4. **Yo-yo Rolling On Inclined Plane**




A yo-yo of mass $m$ has a spool of radius $R$ and an axle of radius $R/3$. Its moment of inertia about an axis passing through the center of the yo-yo can be approximated by $I_0 = (1/2)mR^2$. The yo-yo is placed upright on an inclined plane that makes an angle $\phi$ with respect to the horizontal, and the string is pulled with a force of magnitude $F$ in the direction shown in the figure. The axis of rotation of the yo-yo is perpendicular to the page. The downward acceleration of gravity is $g$. The coefficient of static friction between the yo-yo and the inclined plane is $\mu_s$.

What is the maximum magnitude of the pulling force, $F$, for which the yo-yo will roll without slipping? Express your answer using some or all of the following variables: $\mu_s$, $\phi$, $m$, $R$ and $g$.

5. **Billiards**




A spherical billiard ball of uniform density has mass $m$ and radius $R$ and moment of inertia about the center of mass $I_{cm} = \frac{2}{5}mR^2$. The ball, initially at rest on a table, is given a sharp horizontal impulse by a cue stick that is held an unknown distance $h$ above the centerline (see diagram above). The coefficient of sliding fiction between the ball and the table is $\mu_k$. The average force due to the impulse is much larger than the force of friction, so you may ignore the friction during the time the impulse is applied. The ball leaves the cue with a given speed $v_0$ and an unknown angular velocity $\omega_0$. Because of its initial rotation, the ball starts to skid, but eventually acquires a maximum speed of $\frac{9}{7}v_0$ when it starts rolling without slipping. Find the ratio $h/R$. Your answer should simplify to a simple numerical fraction.